The Rational Formula from the Runhydrograph for Design Flood Estimation

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Abstract

The rational formula, which is possibly the simplest flood estimation technique available, was reviewed in an attempt to extend its use beyond a simple "check" formula limited to small catchments (<100km^2). In order to achieve this, the rational formula needs to be applied in a probabilistic manner, where the variables *c* and *i* (the runoff coefficient and rainfall intensity respectively) of the formula are associated with a probability of exceedence, *T*. The main difficulty in applying the rational formula in this manner is the determination of the formula's runoff coefficient, *c*. The "data set" of runhydrographs, produced by Hiemstra and Francis (1979), was used to calibrate the coefficients in order to obtain objective estimates of *c* for use in a probabilistic manner. It was found that the calibration on this data set of flood peak and volume pairs yielded c_T-coefficients that were similar to those offered by Chow et al. (1988). This independent corroboration justified the use of the latter set of coefficients in large catchments as well as small. It was also discovered that the time base-length of the triangular hydrograph approximating the most probable runhydrograph is approximately 2.25 times the catchment's time of concentration, T_c. The conclusion offers a simple method (to be used amongst others), based the probabilistic use of the rational formula, to obtain complete design flood hydrograph estimates for use at ungauged sites for any sized catchment.

Keywords: *design flood estimation, probabilistic rational formula, runhydrograph, calibration of runoff coefficients.*

1 Introduction

The rational formula was first proposed by the Irish engineer Mulvaney in 1851 and has since become the best known and most widely used method for the determination of peak flood flows from rainfall events. It has survived numerous criticisms regarding its over-simplification of the complex hydrological processes of flood production and is possibly the most favored method used by practitioners for peak flood estimation. The criticisms are based on the following simplifying assumptions, namely the duration of input rainfall required to obtain the maximum rate of runoff is equal to the time of concentration of the catchment, the spatial and temporal characteristics of the rainfall is ignored and the runoff coefficient *c*, which is the ratio of the peak rate of runoff to rainfall intensity, suggests a fixed ratio of runoff to rainfall for the entire site, which in reality is not the case. Consequently, the rational formula is limited in its application to urban drainage systems and small rural catchments, of area less than 15km^2 (in South Africa (HRU, 1972)). However, work that has been done, locally by Alexander (2002) and Pegram (2003), and abroad (Institute of Engineers Australia, 1987), has shown that these cautions were too timid and its use may be extended to catchments of size greater than 100km^2.

The focus of this research is on the latter criticism regarding the runoff coefficient *c*. There are typically few sources of data available to guide in the selection of *c*-values. As a consequence, the choices of *c*-values are subjective and it thus becomes the least precise variable of the rational formula (Chow et al. 1988: 497). With this in mind, values of *c* were calibrated on a distribution of past flood events, based on the Runhydrograph method of Hiemstra and Francis (1979), in order to obtain probabilistic and objective estimates of *c*. By utilizing the flood datasets of Hiemstra and Francis (1979), which describe complete flood hydrographs (i.e. peak, volume and shape), the calibration allowed us to obtain, through the rational formula, complete hydrographs for design flood estimation purposes. The apparent shortcomings of the rational formula as a consequence of its assumptions proved not to be an obstacle in obtaining reasonable flood estimates for design purposes for a large range of catchment sizes. The outcome of this investigation is that the rational formula can be used as a valid companion to other available design flood methods for large catchments as well as small.

This paper forms a summary of an article of the same title submitted by the authors to WaterSA, which is currently under revision.

2 Background

2.1 The Rational Formula

In the traditional application of the rational formula, i.e. deterministic (where the actual rate of peak runoff is determined from an observed rainfall event), the value of c is the ratio of the peak rate of runoff to the rainfall intensity. This is calculated as follows (in *SI* units):

$$Q_{peak} = ciA/3.6 \tag{1}$$

where Q_{peak} is the peak rate of volumetric runoff (in m^3s^{-1}), c is the ratio of the peak rate of runoff to the rainfall intensity, I is the intensity of the rainfall event (in $mm.hr^{-1}$) and A is the area of the catchment (in km^2).

In the deterministic application of the rational formula (Eq. 1), the value of c encapsulates such factors such as topography, land use, vegetal cover, soil type and moisture content, that are involved, at the time of the event, in the conversion of the rainfall into runoff. This is distinct to the probabilistic application of the rational formula, where the variables of i and c (the rainfall intensity and the runoff coefficient terms respectively) are associated with a known exceedence probability. In a probabilistic manner, for the estimation of design floods, the rational formula attempts to estimate, for a given probability of exceedence, the magnitude of the peak discharge that would be equivalent to a discharge estimated from a frequency analysis of flood records if a long and representative record were available at that site. There exists no unique combination of catchment conditions and/or parameters to represent a future event in a probabilistic approach as is the case with the deterministic application of the rational formula. In the former application of the rational formula, the estimation of design floods involves the stochastic generation of conditions and parameters for input into the formula so that the probability distributions inherent in the conditions and parameters are captured.

Pilgrim and Cordery (1993) stated that the design situation is exactly suited to the probabilistic approach of the rational formula and has little similarity with the deterministic rational formula, so that the criticisms associated with the deterministic approach are not necessarily valid for the probabilistic design case. Alexander (1990), in his brief from SANCOLD, stated that as the catchment size increases, the value of c becomes more probabilistic than deterministic in its derivation. Furthermore, as mentioned in Alexander (1990), an increase in c with recurrence interval is necessary to accommodate the known effects which also increase with rainfall intensity but are not accounted for in the traditional calculation of c. The main effect, requiring this increase of c with recurrence interval, is that the catchment is likely to be more saturated at the start of a storm with a longer recurrence interval (Rooseboom et al., 1981). This initial saturation caused by pre-event rainfall is the main reason why one can expect to obtain a higher percentage runoff with an increase in the recurrence interval of an event. Alexander (2002) states that in many of the destructive events observed, severe rainfall events were often preceded by above-normal seasonal rainfall.

The rational formula, as used in a probabilistic approach to estimate design floods, has the same form as Eq. (1), but is defined more specifically as:

$$Q_T = c_T\, i_{(T_c;T)} A/3.6 \tag{2}$$

where Q_T is the peak rate of volumetric flow (in m^3s^{-1}) for a recurrence interval (RI) of T-years, c_T is the runoff coefficient for a T-year event, $i_{(Tc,T)}$ is the T-year storm rainfall intensity (in $mm.hr^{-1}$) of duration equal to the time of concentration of the catchment T_c (in hrs), determined from an Intensity-Duration-Frequency (IDF) relationship of design storms, and A is the catchment area (in km^2). The value of c_T purports to transform a T-year design storm $i_{(Tc,T)}$ of duration T_c, on a catchment of area A, into a flood peak Q_T with the same RI as the causative storm.

The derivation of calibrated c_T-values, towards a probabilistic rational formula, is carried out as follows, as explained in Pilgrim and Cordery (1993):

- Where a set of long and reliable record of flood data from a particular catchment exists, a frequency analysis should be carried out on the observed data to determine design values of flood peaks for a range of recurrence intervals.
- A design formula for the calculation of time of concentration T_c must be selected and used consistently throughout the derivation and use of this method. In this study the Kirpich (1940) formula was used:

  $$T_c = 0.0633[L/S]^{0.385} \tag{3}$$

  where T_c is (in hrs), L (in km) is the length of the longest water course and S is the slope of the longest water course.
- Design rainfall intensities, $i_{(Tc,T)}$, for the corresponding time of concentration of the catchment and recurrence interval should be determined from a suitable Intensity-Duration-Frequency (IDF) database. From these data, values of c_T can be back calculated by comparison with the T-year floods for each basin.
- These calibrated values of c_T can then be regressed on any physical characteristic of the catchment. However it was noted by Pilgrim and Cordery (1993) that the probabilistic runoff coefficients determined for Australia did not show much sensitivity to catchment characteristics, contrary to the assumptions that underlie the tabulated design values of c_T as in Chow et al. (1988: 498).

Finally it is important to note that the values of c_T used in this manner are conditioned on the use of a consistent formula for the calculation of T_c and a consistent database for the derivation of the IDF rainfall relationships.

In South Africa, Alexander (2002) proposed a flood estimation technique called the standard design flood (SDF). This method is essentially a probabilistic approach to the rational formula, as advocated by him (Alexander, 1990). The SDF method is based on the calibration of the runoff coefficient against design floods determined from the frequency analysis of recorded events using the log-Pearson-III (LPIII) distribution. The average ratio of Alxander's 50-year SDF flood peak to the 50-year LPIII flood peak was found to be approximately 210% (Görgens, 2002). Alexander (2002) states that the over-estimates fall within the range of uncertainties associated within all design flood procedures. However Görgens (2002), states that although the cost and implications associated with a conscious over-design in terms of a bridge/culvert is relatively minor, by contrast it is not acceptable for dam spillway design, where the cost of the spillway is a significant component of the total dam cost. An average over-estimate of 200% might render some projects infeasible. As such, Görgens recommends that the SDF should be seen as a conservative approach similar to that of the regional maximum flood (RMF) method.

2.2 The Runhydrograph

The runhydrograph method of Hiemstra and Francis (1979) summarizes, for a given catchment, the family of characteristic peak and volume discharges for a given recurrence interval. They were based on the frequency analyses of all rare hydrographs (which were carefully screened for reliability) in a continuous stream flow record and, as such, are independent of rainfall input and catchment characteristics. This set of statistics formed a valid data set against which to calibrate the runoff coefficient towards a probabilistic approach of the rational formula.

The runhydrograph method was developed by Hiemstra and Francis (1979) (in the sequel referred to as H&F) and was based on earlier work by Hiemstra (1972, 1973 and 1974), Hiemstra et al. (1976) and Francis (1979). It is based on the joint probability analyses of same-event flood peak and flood volume pairs of recorded data from 43 catchments around South Africa. H&F discovered that the natural logarithms of the flood peak and its corresponding volume were approximately lognormally distributed and well correlated, with a mean cross-correlation coefficient of 0.78 and a standard deviation of 0.12. Fig. 1 shows the natural logarithms of the recorded flood peak (*U*) and volume (*V*) pairs plotted in probability space together with the contours of equal probability of a standardized bivariate normal probability density function (with a cross-correlation coefficient of 0.85). Also shown in Fig. 1 (in the positive quadrant) are 10- and 100-year return period exceedence probability contours (bold lines). The dashed lines intersecting on the 100-year exceedence contour define a quadrant to the upper right in the plane whose probability density integrates to 0.01. Thus, on average, 1% of the observations will lie within this area, and within other areas defined similarly on the 100-year contour.

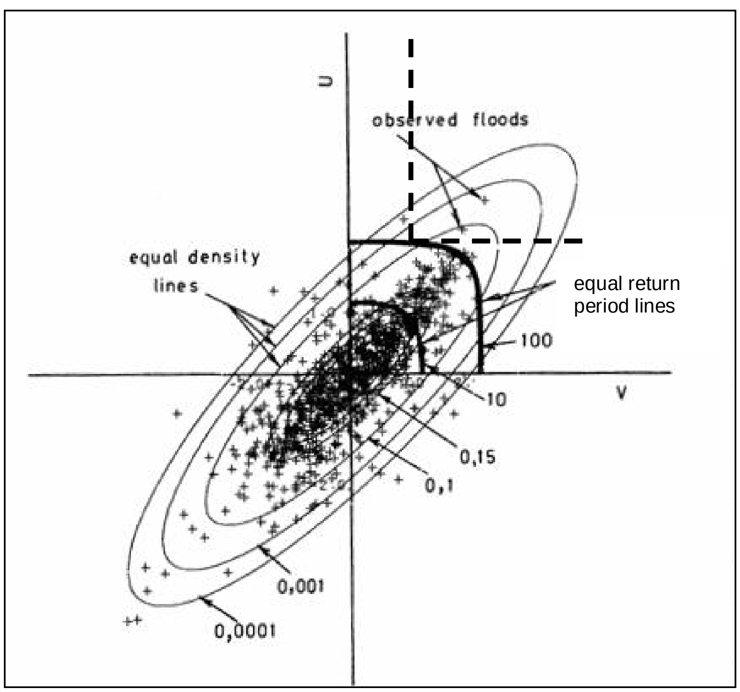

Figure 1: A standard bivariate normal probability density function, with a cross correlation coefficient of 0.85, plotted with log-transformed observed flood peak-volume pairs in probability space (Hiemstra and Francis, 1979: 14). The bold lines in the positive quadrant are the 10- and 100-year return period joint-exceedence contours. The dashed lines include a quadrant to the upper right which, on average, will include 1% of the observations.

The exceedence contours describe the joint probability of flood peak and flood volume exceedence and are able to produce "families" of hydrographs (peak-volume pairs) of equal probability of jointly being exceeded, but of varying shape. These families can range from the marginal peak (associated with any volume), to the "most likely" joint peak and volume pair through to the marginal volume, each with an equal probability of joint exceedence. However, it can be seen from Fig. 1 that the plotted peak-volume pairs cluster around the 45° line in an elliptical shape. If the cross-correlation coefficient approaches unity, the minor axis of the ellipse reduces to zero. Thus, although more than one combination of a peak-volume pair exists that has the same probability of jointly being exceeded, the most likely pair will be found at the intersection of the 45° line and the exceedence contour where the probability density is highest.

Fig. 2 shows the application of the runhydrograph method for design flood peak and volume estimation for a cross-correlation coefficient of 0.85. The listed numbers on the top right of Fig. 2 are the standardized ordinates of the peak-volume exceedence contours for the selected recurrence intervals. They describe the joint exceedence of the most likely peak-volume pair (corresponding to line #1) through to the exceedence of the marginal peak (corresponding to the vertical axis to the left of line #6). It is unlikely that a peak-volume pair will occur on lines 4, 5 and 6 for this relatively high correlation, and thus for the purposes of this investigation the most likely peak-volume pair was used to limit the number of variables.

This idea, of describing hydrographs with a joint probability of peak and volume exceedence, has surfaced again more recently to be exploited in the evaluation of dam safety (De Michele et al., 2005).

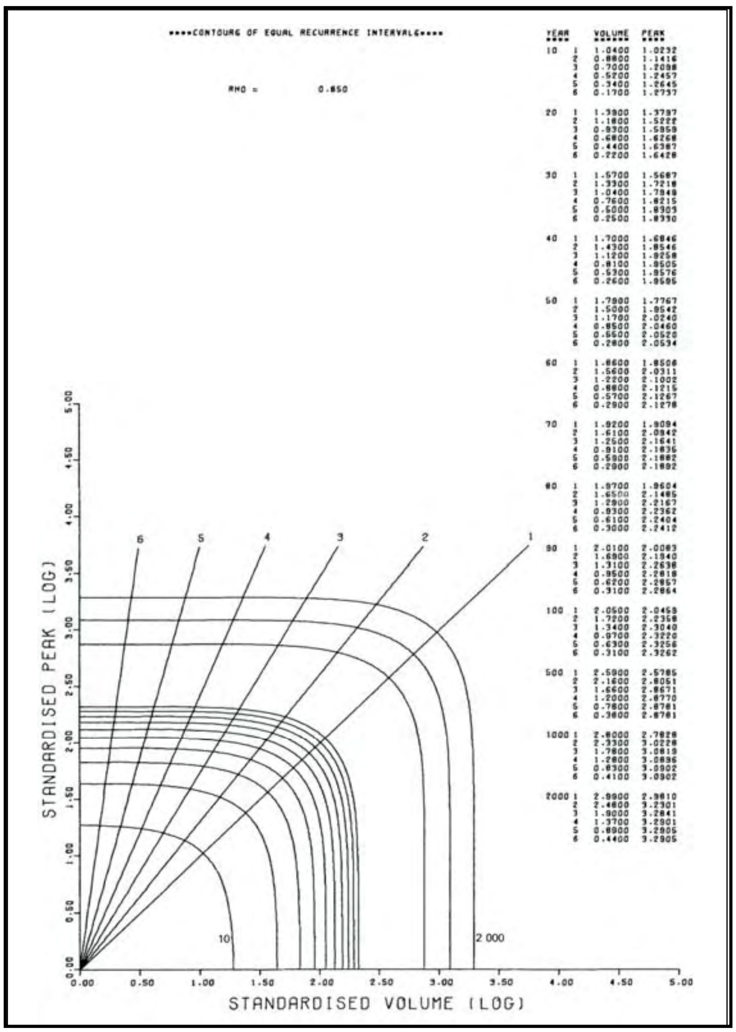

Figure 2: Joint flood peak and flood volume exceedence contours, in probability space for a peak-volume cross-correlation coefficient of 0.85 (Hiemstra and Francis, 1979: 53).

3 Investigation

3.1 The Rainfall Database

Design rainfall estimates were obtained from Smithers and Schulze (2002) design rainfall database for South Africa. This database provides point rainfall depths (in mm) for storm durations ranging from 5 minutes to 7 days and for return periods of 2 to 200 years at a spatial resolution of 1 arc minute in South Africa. A computer program, with a graphical user interface, has been developed to obtain design rainfall depths for any location in South Africa from this database. The software may be downloaded from the following website: http://www.beeh.unp.ac.za/hydrorisk/ by following the "Design Rainfall" option.

The mean precipitation depth for each catchment was computed by averaging a number (depending on the size of the catchment) of point estimates along the main watercourse. This method was used to account for the variation in precipitation with position and altitude for large catchments. Area reduction factors (ARFs) were deemed not necessary based on the findings of Pegram (2003), who found that the point rainfall is automatically scaled by a constant ARF when the duration of the input storm rainfall is equal to the catchment's time of concentration T_c, as demanded by the rational formula. T_c was determined from Kirpich's formula (1940) in Pegram's study and the same formula is consequently used here.

By fitting a simple power-law function of storm duration to the mean T-year rainfall intensity for the determination of the intensity, duration and frequency (IDF) relationships, where i_T (T-year rainfall intensity in mm.hr^{-1}) = ad^{-c} (d is the storm duration (in hrs) and a and c are the fitted power-law parameters), it was found that rainfall intensity scaled, for all catchments studied, on average to the power of *-0.762* of rainfall duration with a standard deviation 0.0419.

3.2 Hydrograph Shape from the Streamflow Database of the Runhydrograph Method

Hydrograph shape is described in terms of the flood peak Q, flood volume V and time base-length B. The "most likely" T-year peak and volume pair, Q_T and V_T respectively, were computed for each of the study catchments and for a range of T using the runhydrograph method of H&F. These were combined with catchment parameters, such as area (A) and time of concentration (T_c - based on Kirpich's formula), taken from Petras and du Plessis (1987). Through simple geometric calculations, the hydrograph base-length B was determined from the following relationship: $V = 0.5B \times Q$. The average ratio of ${}^{B}/_{T_c}$ for all catchments (except 3 which gave spurious results) was then determined for each recurrence interval. The results are presented in Table 1 together with their standard deviations.

Table 1. The average ratio of the hydrograph time base-length *B* to the catchment's time of concentration T_c for all catchments. The standard deviations for the averages in each recurrence interval is also given.

Recurrence Interval, *T* (years)	10	20	50	100	200
Mean of ${}^{B}/_{T_c}$ ratios	1.92	2.06	2.25	2.40	2.56
Standard deviation	0.98	1.09	1.29	1.48	1.71

Several points are worth noting. Firstly the base-lengths are, on average, 2.25 times larger than the catchments' time of concentration across all recurrence intervals. Secondly, the tendency of the base-length to increase with T is likely to be due to the method employed by H&F in extracting their hydrographs. As depicted in Fig. 3, H&F employed a truncation level for each catchment in order to extract independent hydrographs from their continuous records of stream flows. Flood volumes were obtained by extrapolating the rising limb and the recession limb of the discharge curves downwards towards zero flow from the first point below the truncation level which showed a reversal in slope. Depending on this level, a higher truncation level is likely to result in a reduction in the modeled volume when compared to the actual volume of the flood event. This accounts for the trend in Table 3.

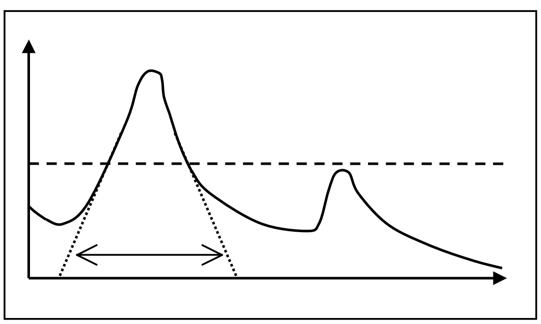

Figure 3. The method employed by Hiemstra and Francis (1979) to extract independent hydrographs from a continuous flow record, showing that a lower truncation level is likely to provide a bigger volume.

3.3 Calibration of *c*

The *T*-year flood peaks from H&F were combined with the representative *T*-year rainfall intensities for the corresponding catchments. Calibrated values of c_T were then back calculated based on the following equation:

$$c_T = Q_T \times 3.6 / \left(i_{(T_c;T)} \times A \right) \tag{4}$$

where the variables are as defined for Eq. 2. The coefficients calibrated in this study were similar in magnitude to *c*-coefficients tabulated in Chow et al. (1988: 498). The coefficients listed in in this source (reproduced in Table 2 below) are a function of land use type, slope and recurrence interval and were designed to be used for small catchments only (i.e. less than 100km^2). Fig. 4 shows the comparison of the calibrated c_T-coefficients with those listed in Chow et al. by plotting the coefficients from the two sources coaxially as a function of recurrence interval. It can be seen in the figure that the two sets of coefficients are of the same order of magnitude with the *c*-values from Chow et al. being generally more conservative. The figure also shows that growth of the calibrated coefficients, with recurrence interval, is on average gentler than the tabulated coefficients of Chow et al.

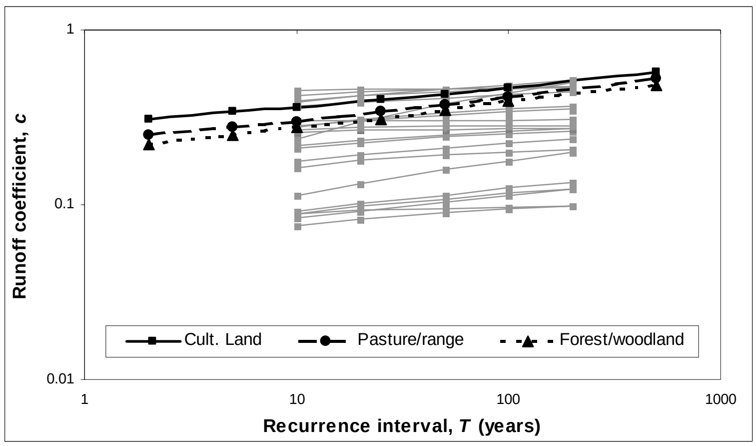

Figure 4. A comparison of the runoff coefficients, *c* from Chow et al. (1988: 498) with those calibrated in this study, c_T. The *c*-values from Chow et al. are shown in thick bold lines and correspond to the three land coverage types (all of "flat slopes"), namely "cultivated land", "pasture/range" and "forest/woodland" (see Table 2). The calibrated coefficients achieved in this study are coaxially plotted (thin, grey lines) for each catchment.

Table 2. Runoff coefficients for use in the rational method for undeveloped (rural) regions for Austin Texas in the U.S. (from Chow et al., 1988: 498).

	Runoff coefficients *c*							
Character of Surface (undeveloped)	**2-year**	**5-year**	**10-year**	**25-year**	**50-year**	**100-year**	**200-year** (interpolated)	**500-year**
Cultivated Land								
Flat, 0 - 2%	0.31	0.34	0.36	0.40	0.43	0.47	0.51	0.57
Average, 2 - 7%	0.35	0.38	0.41	0.44	0.48	0.51	0.55	0.6
Steep, >7%	0.39	0.42	0.44	0.48	0.51	0.54	0.57	0.61
Pasture/Range								
Flat, 0 - 2%	0.25	0.28	0.3	0.34	0.37	0.41	0.46	0.53
Average, 2 - 7%	0.33	0.36	0.38	0.42	0.45	0.49	0.52	0.58
Steep, >7%	0.37	0.4	0.42	0.46	0.49	0.53	0.56	0.6
Forrest/Woodlands								
Flat, 0 - 2%	0.22	0.25	0.28	0.31	0.35	0.39	0.42	0.48
Average, 2 - 7%	0.31	0.34	0.36	0.4	0.43	0.47	0.50	0.56
Steep, >7%	0.35	0.39	0.41	0.45	0.48	0.52	0.54	0.58

3.4 Verification of c_T

The aim of verification tests was to determine whether the calibrated coefficients c_T could be applied in a probabilistic manner to estimate design floods that were equivalent to floods determined from a statistical analysis of observed events at the site. Statistically modeled flood peaks (using a general extreme value (GEV) distribution) were compared to flood peaks, for the corresponding catchments, determined from the rational formula through the use of the coefficients calibrated in this study. Thus the area and rainfall intensity variables were as before. The results of these tests are shown below in Fig. 5 (for the 10-year case) and summarized in Table 3 for all recurrence intervals, where Q_{RF} are the flood peaks determined from the rational formula through the use of calibrated coefficients and Q_{GEV} are the statistically modeled flood peaks using the GEV distribution.

An important issue needs to be mentioned at this juncture. The coefficients calibrated in this study were based on floods that have a joint peak and volume exceedence, and consequently the floods predicted through the use of these coefficients,have a joint probability exceedence. The statistically modeled flood peaks were based on a marginal probability of flood peak exceedence only. The magnitude of this difference can be estimated from the exceedence contours of Fig. 2, and is the difference between the "most likely" flood peak and volume pair and the marginal flood peak..This translates into an average magnitude difference of 0.7 (across the recurrence interval range of 10- to 200-years and for an average cross correlation coefficient of 0.85), where the magnitude of the joint peak exceednce is 0.7 of the magnitude of the marginal peak.

Table 3. A summary of the trend-line fits for the graphs of Q_{RF} vs. Q_{GEV} from Fig. 5, where Q_{RF} are the flood peaks determined from the rational formula through the use of calibrated coefficients, Q_{GEV} are the statistically modeled flood peaks using the GEV distribution, *a* is the slope of the trend-line fitted to the data and R^2 describes the accuracy of the fit.

Recurrence Interval *T* (in years)	10	20	50	100	200
a	0.970	0.954	0.940	0.932	0.925
R^2	0.890	0.885	0.844	0.780	0.684

The results of Table 5 indicate that the relationship between the probabilistically used rational formula Q_{RF} and the statistically modeled floods Q_{GEV} are very close to being linear (with an average slope *a* of 0.944). The accuracy of the fits are good as well. It was also found that the average ratio of Q_{RF}/Q_{GEV} was approximately *0.70*, which was as expected. This translates into a drop of 30% for all the catchments and across all recurrence intervals and reflects the difference between the marginal peak and the joint peak-volume pair. The results of this verification indicate that the c_T-coefficients obtained in the calibration process are as expected and perform the job for which they were derived, i.e. to predict probabilistically determined floods for large catchments.

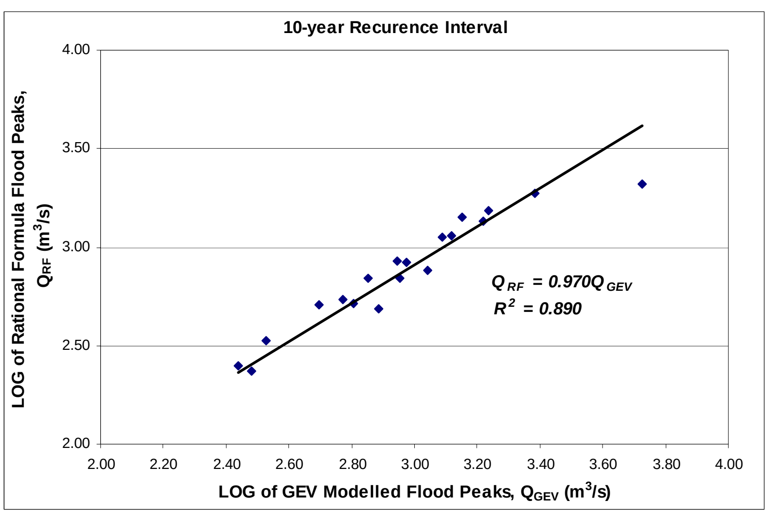

Figure 5. A comparison of the rational formula flood peaks Q_{RF} and the GEV modeled flood peaks Q_{GEV} for the 10-year recurrence interval for the purposes of verification. Q_{RF} was computed in a probabilistic manner using the rational formula (with calibrated coefficients) and Q_{GEV} comprised of the 10-year floods estimated from a frequency analysis (using a GEV distribution) of recorded events at the sites. The large outlier to the right of the figure pulls the relationship away from unity.

3.3 Validation of c_T

In order to validate the calibrated c_T-coefficients, it was necessary to find some regional descriptor(s) on which to regress the coefficients. This was required so that the probabilistic coefficients might be extended to ungauged catchments.

Several regional descriptors were tested in combination with the $c_{(T)}$-values to examine if a relationship existed on which to regress the coefficients. Descriptors such as catchment slope, mean annual precipitation (MAP), percentages of land coverage and Kovaćs' regional *K*-values (Kovaćs, 1988) were tested. From these analyses, no meaningful relationships between any of the descriptors tested and the $c_{(T)}$-coefficients were found. There were also no relationships found between parameters (multiplier and exponent) of a power-law function fitted to the $c_{(T)}$-values as a function of recurrence interval and regional descriptors. This result is in line with the comments of Pilgrim and Cordery (1993) for conditions in Australia, where the probabilistic runoff coefficients did not show much sensitivity to catchment characteristics.

However the c_T-coefficients achieved in this study did show a reasonable similarity to the values (reproduced in Table 2) listed in Chow et al. (1988: 498). It was decided to use these tabulated *c*-values from Chow et al., as default values, to test the probabilistic rational formula in validation as, in spite of considerable effort, it was not possible to find a meaningful relationship between c_T and any regional parameters.

Catchments, which were not used in the calibration nor verification parts of the study were identified for which flood records were available and modeled in Pegram and Parak (2004) using a GEV distribution. These ranged in size from 126 km^2 to 24 044km^2. For these catchments, times of concentration T_c-values, land coverage percentages and standard slopes were obtained from Petras and du Plessis (1987). Design rainfall intensities were obtained from the Smithers and Schulze (2002) rainfall database in the same manner as before.

The land coverage types catalogued in Petras and du Plesis were based on the following types: forest, dense bush wood, thin bush wood, cultivated land, grass and bare. In order to correlate this catchment type with the generalized coverage types in given in Table 2 (from Chow et al. (1988: 498)), several assumptions were made. They were: 1) that the greatest percentage of land coverage (the modal type) was representative of the entire catchment and 2) certain descriptions of coverage types from the two sources (of Petras and du Plessis and Chow et al. respectively) were equivalent (shown in Table 4). It should be noted that all catchments used in the validation tests had slopes that were less than 2% (i.e. they were of the "flat slopes" type). Secondly, the correlation of the catchment's catalogued land coverage type (of Petras and du Plessis) with that of the generalized land coverage type (of Table 2) was made subjectively and had the tendency to be coarse. However, since no exact method to relate the two description types existed, the procedure described above was adopted. The results of this exercise did not show any gross irregularities and supported the method employed, proving the *c*-values listed in Table 2 to be quite robust.

Table 4. Equivalent land coverage types from the descriptions of Petras and du Plessis (1987) and Chow et al. (1988: 498).

Equivalent land coverage types	
Actual catchment land coverage (as described in **Petras and du Plessis** (1987))	*c*-coefficient land coverages (as described in **Chow et al.** (1988: 498))
Forest	Forest/Woodland
Dense Bush Wood	Forest/Woodland
Thin Bush Wood	Forest/Woodland
Cultivated Land	Cultivated Land
Grass	Pasture/Range
Bare	Cultivated Land

Design flood peaks were obtained using the rational formula method in a probabilistic manner, but with the runoff coefficients from Table 2 substituted for the calibrated coefficients. These design flood peaks Q_{RF} were compared with statistically modeled flood peaks Q_{GEV}, of the same catchments, for the corresponding recurrence intervals. The results of validation for the 10-year recurrence interval are shown in Fig. 6 and the results for all recurrence intervals are summarized in Table 5.

Table 5. A summary of the trend line fits for the graphs of Q_{RF} vs. Q_{GEV} (from Fig. 6) where *a* is the slope of the trend line fitted to the data and R^2 describes the accuracy of the fit. Q_{RF} was determined using the *c*-values listed in Chow et al. (1988: 498).

Recurrence Interval, *T* (years)	10	20	50	100	200
a	1.07	1.05	1.03	1.01	0.997
R^2	0.656	0.669	0.656	0.626	0.572

The results in Table 5 indicate that the relationship between the rational formula flood peaks Q_{RF} (utilized in a probabilistic manner with the *c*-values from Chow et al., 1988: 498) and the statistically modeled floods Q_{GEV} are very close to being linear (with an average slope *a* of 1.03). However the accuracy of the fits is not great, though they are reasonably encouraging. The significance of these results is that the *c*-values from Chow et al. (1988: 498) perform well as a substitute for the calibrated coefficients in a probabilistic manner for catchments of varying size with an accuracy of about 30%.

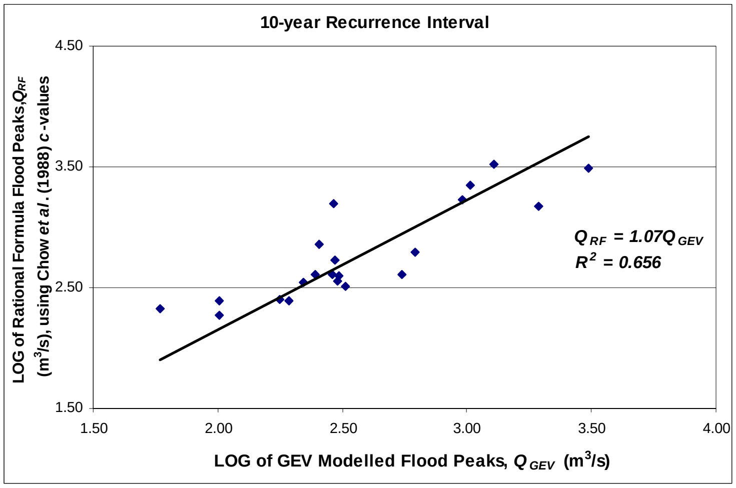

Figure 6. A comparison of the rational formula flood peaks Q_{RF} and the GEV modeled flood peaks Q_{GEV} for the 10-year recurrence interval for the purposes of validation. Q_{RF} was computed in a probabilistic manner using the rational formula with the *c*-coefficients from Chow et al. (1988: 498) substituted for calibrated coefficients. Q_{GEV} comprised of the 10-year flood peaks estimated from a statistical analysis of recorded events at the sites.

A note on the relative accuracy of the rational formula variables is appropriate to explain the accuracy achieved in validation. For example, for an RI of 50-years, the *c*-values from Chow et al. (1988: 498) have a median value of 0.45 and range from 0.35 to 0.51, across all land coverage types and catchment slopes (see Table 2). In percentage terms, this translates into a lower and upper difference of -28% and +13% respectively about the median. The accuracy with which the design storm intensity *i* is measured depends on the applicability of the rainfall database at the site of the catchment concerned, as in any rainfall-runoff calculation, but more crucially on the calculation of the storm duration, *d* (which equals T_c for the rational formula). This is because the intensity varies as $d^{-0.762}$ on average in South Africa (shown previously), so an error of 20% in *d* produces a concomitant error of 18% in the intensity variable. Although there may be objections to the use of the simplest estimate of T_c, as suggested by Kirpich (1940), it should be noted that its *consistent* usage in the rational formula would maintain the level of precision. The remaining variable, catchment area, can of course be calculated to any precision desired.

The result of this is that the implicit error in the rational formula calculation of floods is likely to be of the order of 36%. This figure was calculated as the standard deviation of the product of two independent normally distributed (N(0,1)) random variables with standard deviation’s of 28% and 18% (modeling the errors in the coefficient with an RI of 50-years and the duration respectively). When a regression line was fitted to this set of simulated data, an R^2 was 0.8 was achieved. The R^2 value depends on the range of flood peak magnitudes where the largest flood used in validation was 30 times the smallest. Thus the 36% “precision” is probably of the same order as that with which the larger flood-peaks are measured and the results of the validation tests reflect this.

What is encouraging is that the sites used in validation ranged in size from small to large catchments (126 to 24 044 km^2). As a result, it can be cautiously concluded that the *c*-values from Chow et al. (1988: 498) can be applied in a probabilistic manner (equivalent to calibrated coefficients) for any sized catchments in this region, as a check calculation for flood prediction.

4 Conclusion

This work has shown that the use of the rational formula on any sized catchment, as one of a suite of estimation methods, is valid provided the estimation of the runoff coefficient is probabilistic in nature. It was shown that the *c*-coefficients calibrated in this study yielded c_T-coefficients that were similar in magnitude to those offered by Chow et al. (1988: 498). A validation of this observation justified the use of the latter set in large catchments as well as small. A further derivation arising from this investigation was that the time base-length of the triangular hydrograph, approximating the runhydrograph peak-volume pair, was found to equal to 2.25 times the catchment's time of concentration (T_c). Based on these findings, a *design flood estimation algorithm*, to be used among other methods, can be cautiously postulated in the six steps below for any sized catchment in South Africa. It is important to note that these findings are conditioned on the consistent use of Kirpich's (1940) formula for the calculation of the catchment's time of concentration T_c and the South African rainfall database of Smithers and Schulze (2002).

1. Identify the catchment.

2. Map the catchment's morphometry: obtain the effective catchment area, longest water course and the slope of longest water course.

3. Use the Kirpich (1940) formula for the catchment's time of concentration (from Eq. 3)

$$T_c = 0.0633[L / S]^{0.385} \tag{3}$$

4. Use Smithers and Schulze (2002) to obtain a design rainfall estimate. An average rainfall depth estimate for the catchment should be obtained, for the selected RI, by averaging a few estimates of depth along the main water course of the catchment. A power law relationship $i \sim d^{-0.762}$ should be fitted to obtain the intensity for the duration of T_c. Modification of this intensity by an ARF is not necessary.

5. Obtain the percentages of land coverage and take the modal type as being representative of the entire catchment. Table 2 should be consulted to obtain the *c*-values as a function of the selected RI, land coverage type and slope of the catchment.

6. Compute a probabilistic flood peak estimate from the rational formula (from Eq. 2):

$$Q_T = c_T \, i_{(T_c;T)} A / 3.6 \tag{2}$$

This peak has a joint probability of peak and volume exceedence. If only the marginal peak estimate for a given T is required, then factor $Q_{(T)}$ up by 1.43. The approximate triangular hydrograph for the joint peak and volume is defined as follows: Time to peak is equal to T_c and the hydrograph base length B is equal to $2.25T_C$.

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